Direct Numerical Simulation
Large Eddy Simulation

TURBULENT FLOWS AND INHERENT STRUCTURES

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Introduction to Turbulent Flows

- Most flows encountered in engineering practice are Turbulent
- Turbulent Flows are characterized by the fluctuating velocity field (both position and time). We say that the velocity field is Random.
- Turbulence highly enhances the rates of mixing of momentum, heat etc…

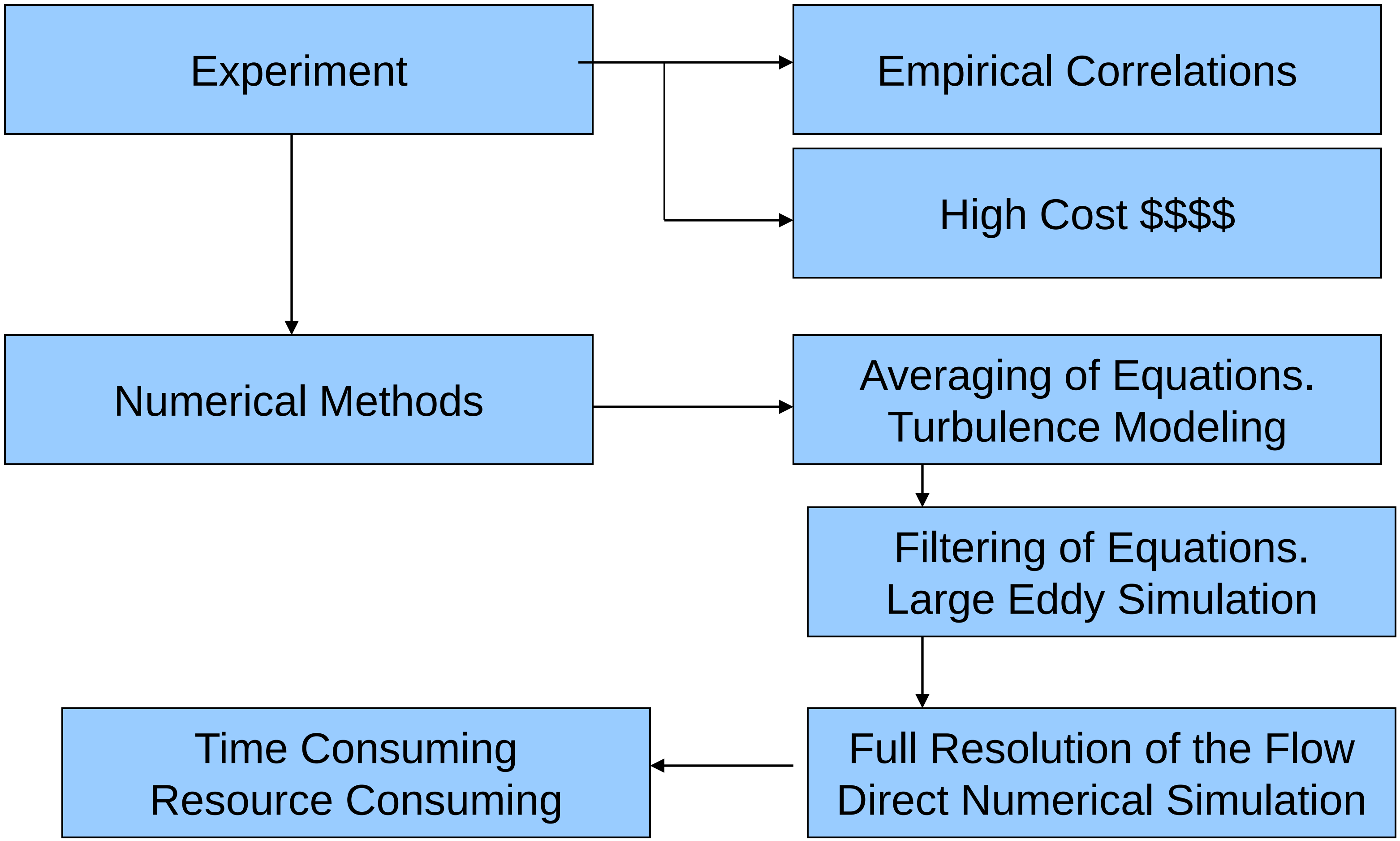
Introduction to Turbulent Flows

- The motivations to study turbulent flows are summarized as follows:
  - The vast majority of flows is turbulent
  - The transport and mixing of matter, momentum, and heat in turbulent flows is of great practical importance
  - Turbulence enhances the rates of the above processes

Introduction to Turbulent Flows

- The primary approach to study turbulent flows was experimental
- With the increase of precision and sophistication of eng' applications, the experiments are no more efficient
- Therefore, more effort was directed towards the numerical solution of the flow equations.

Introduction to Turbulent Flows



Governing Equations

- Continuity Equation:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{U}) = 0$$

- Momentum Equations:

$$\frac{D\boldsymbol{U}}{Dt} = -\frac{1}{\rho}\nabla p + \nu \ \nabla^2 \boldsymbol{U}$$

Random Fields

- In a Turbulent flow, the velocity field is said to be RANDOM. What does that mean? Why is it so?
- Consider a Fluid Flow experiment that can be repeated several times under the same set of conditions

Random Fields

- Assume you want to measure a component of the velocity field U(x1,t1)
- Consider the event that A={U(x1,t1)<10m/s}
  - If A inevitably occurs, then A is certain
  - If A cannot occur, then A is impossible
  - Another possibility is that A may but need not occur, then A is Random

Random Fields

- The word Random does not hold any sophisticated significance as it is usually assigned.
- The event A is random means only that it may or may not occur

Random Fields

- Below is the measured velocity at 40 repetitions of the experiment

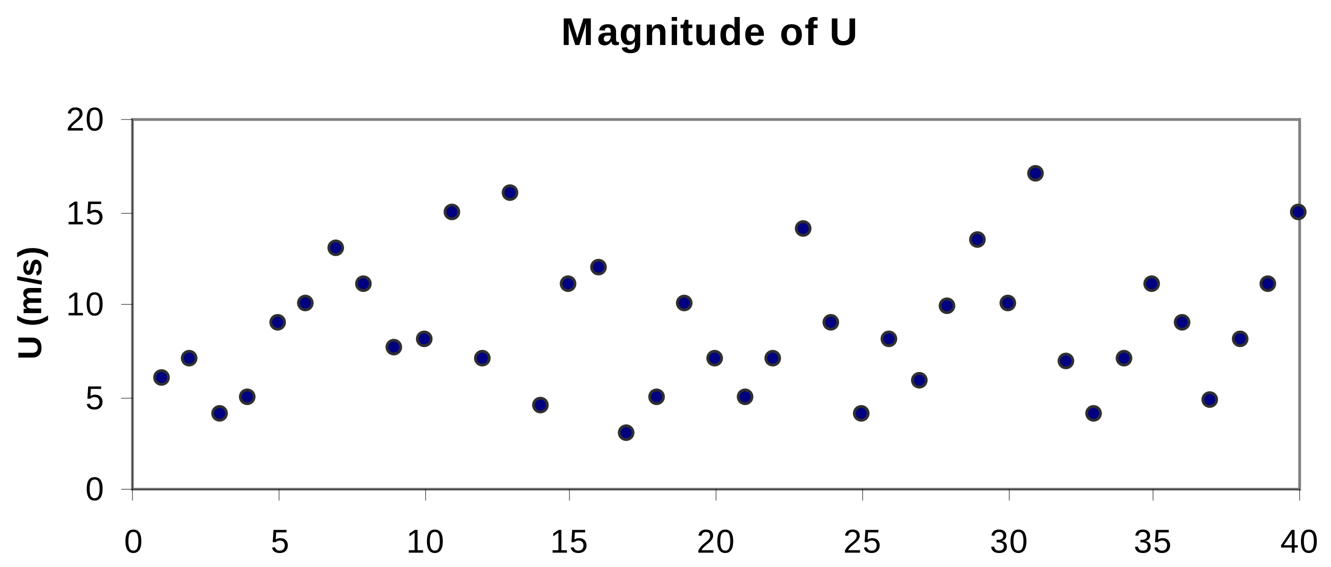


Figure 1.1: Magnitude of U as a function of Experiment Number

Random Fields

- The cause of this are the initial or boundary conditions of the experiment. It can be shown that a dynamic system governed by certain PDE's prohibits very acute responses to tiny variations in boundary conditions.
- Why doesn't this happen in a laminar flow? Because of the Reynolds number.
- Example: Lorentz dynamic system

Random Fields

- The Lorentz dynamic system is a typical example of this sensitivity.

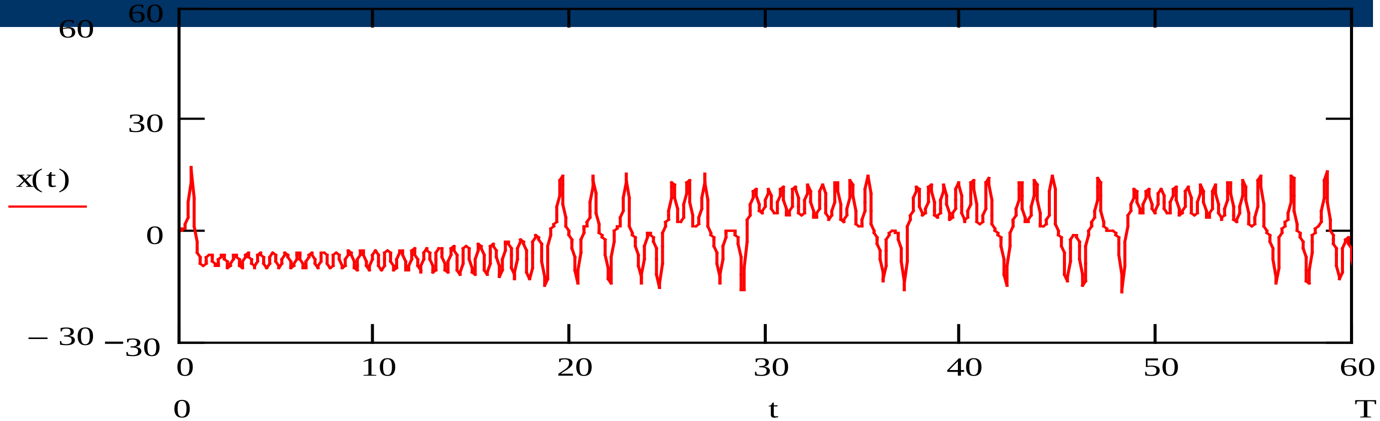
$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = \rho x - y - xz$$
$$\dot{z} = -\beta z + xy$$

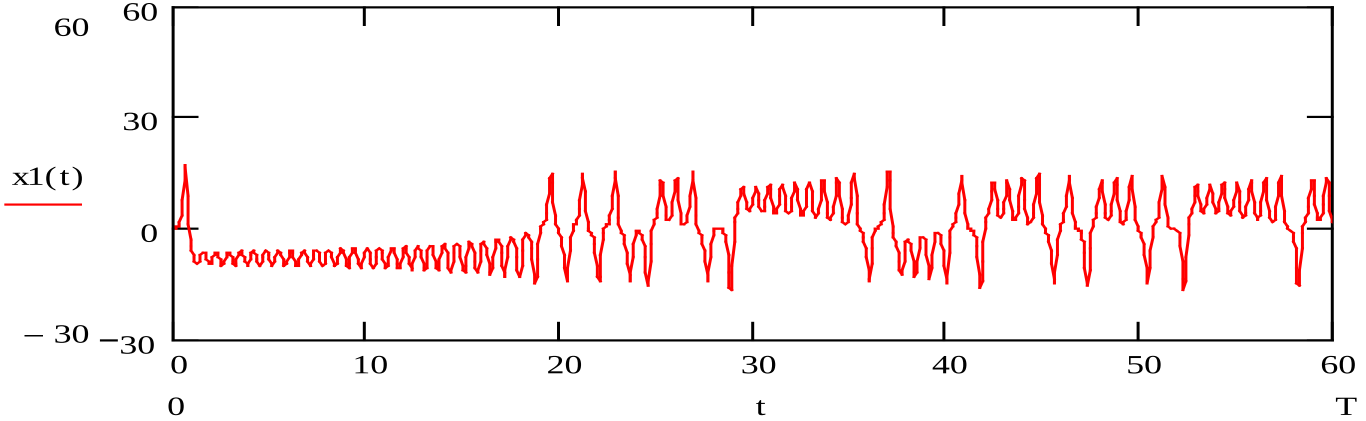
With two sets of initial conditions as follows:

[x(0),y(0),z(0)]=[0.1,0.1,0.1] and

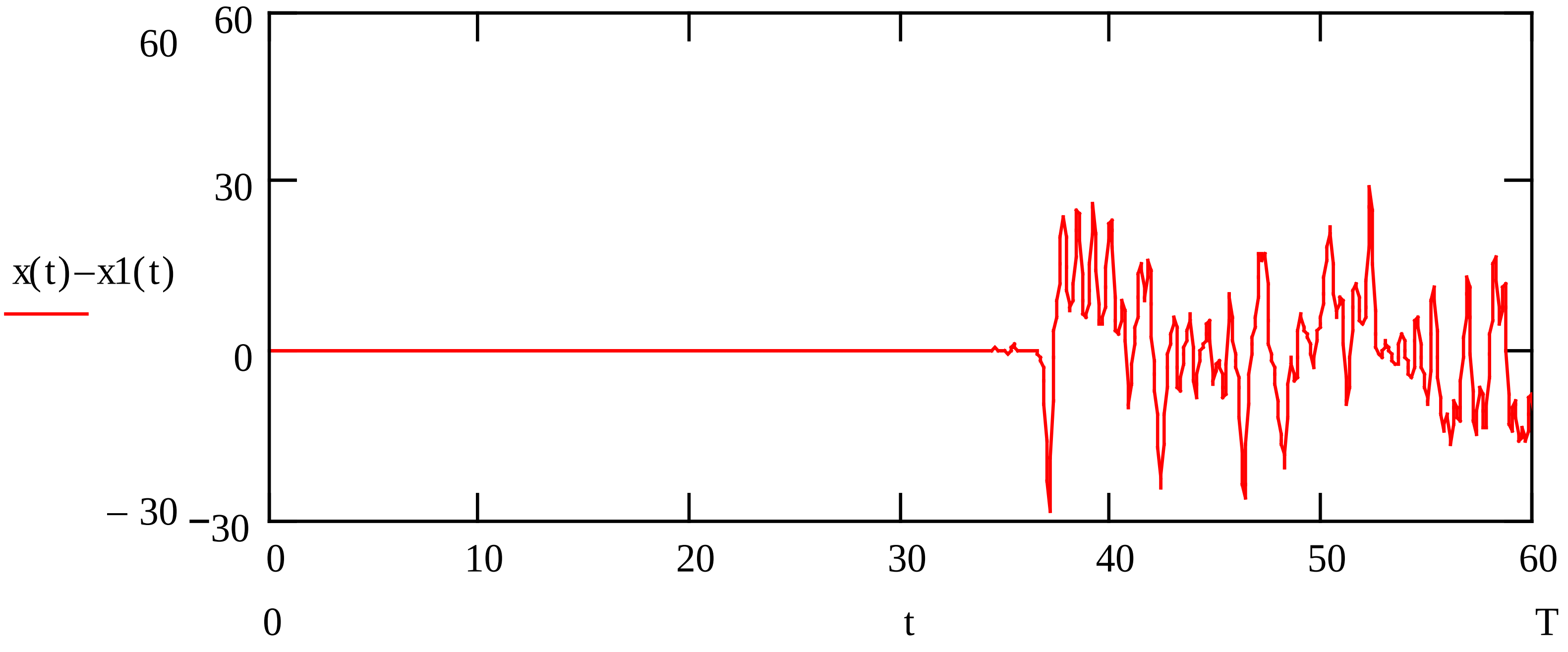
[x1(0),y(0),z(0)]=[0.1000001,0.1,0.1]

Random Fields





Random Fields



Random Fields

- As we can see, the figures show the sensitivity of the system. In fact, for a critical value of the coefficients (fix σ, β) say ρ =24, the system becomes highly random.
- This coefficient corresponds to the reynolds number in fluid flow. Beyond a critical value, the flow becomes random, i.e. turbulent.

The Energy Cascade Mechanism

- Turbulent flows are characterized by an infinite number of time and length scales.
- This can be shown by the hypothesis of the energy cascade mechanism presented by Richardson in 1922
- Turbulence can be considered to be composed of eddies of different sizes

The Energy Cascade Mechanism

- An eddy is considered to be a turbulent motion localized within a region of size l
- These sizes range from the Flow lengthscale L to the smallest eddies.
- Each eddy of length size ℓ has a characteristic velocity u(l) and timescale t(l)=u(l)/l
- The largest eddies have lengthscales comparable to L

The Energy Cascade Mechanism

- Each eddy has a Reynolds number
- For large eddies, Re is large, i.e. viscous effects are negligible.
- The idea is that the large eddies are unstable and break up transferring energy to the smaller eddies.
- The smaller eddies undergo the same process and so on

The Energy Cascade Mechanism

- This energy cascade continues until the Reynolds number is sufficiently small that energy is dissipated by viscous effects: the eddy motion is stable, and molecular viscosity is responsible for dissipation.

The Energy Cascade Mechanism

- Big whorls have little whorls,
  which feed on their velocity;
  and little whorls have lesser whorls,
  and so on to viscosity

The Energy Cascade Mechanism

- What is the size of the smallest eddies?
- As l decreases, do u(l) and t(l) decrease?
- The above questions are answered by the Kolmogorov hypotheses

The Kolmogorov Hypotheses

- Local Isotropy: at sufficiently high Re, the small scale turbulent motions are statistically isotropic.
  As the energy passes down the cascade, all information about the geometry of the large eddies (determined by the flow geometry & BC) is also lost. As a consequence, the small enough eddies have a somehow universal character, independent of the flow.

The Kolmogorov Hypotheses

- First Similarity: In every turbulent flow at very high Re, the statistics of the small scale motions are universal and uniquely determined by ε and ν. The smallest eddies that are contained in the dissipation range are affected by ε and ν.

The Kolmogorov Hypotheses

- Second similarity: at sufficiently high Re, there is range of small eddies, smaller than the flow scale yet larger than the smallest eddies, and these are little affected by viscosity because they have a high enough Reynolds number.

Direct Numerical Simulation

- In DNS, all the length and time scales are resolved.
- We make direct use of the NS equations
- A DNS is equivalent to a lab experiment
- The data calculated is more than enough for engineering purposes.
- DNS is highly informative regarding the physics of fluid flow

Direct Numerical Simulation

- However, Computer cost (memory, CPU time, hardware…) increases with the Reynolds number
- The grid should be as fine as possible, and in each direction, the number of nodes is proportional to Re^(3/4), so, for a general 3D flow, the total number of nodes is proportional to Re^(9/4)

Direct Numerical Simulation

- Therefore, DNS is currently applied to simple flows such as channel flows and free shear flows.
- Although DNS is the simplest from numerical point of view, the discretized equations also need special treatment in that finite difference techniques (and the other standard techniques) cannot be used.

Direct Numerical Simulation

- In DNS, we use what is called spectral methods, in that we express the velocity field as a Fourier series (in spectral space) and the procedure is then to calculate the coefficients of the fourier series.

$$\mathbf{u}(\mathbf{x},t) = \sum_{\boldsymbol{\kappa}} e^{i\kappa\cdot x}\hat{\mathbf{u}}(\kappa,t)$$

Direct Numerical Simulation

Re_L	N (# of nodes in each direction)	N^3 (total nodes)	M (time steps)	CPU	Time
94	104	$1.1x10^6$	$1.2x10^3$	20	Min
375	214	$1.0x10^7$	$3.3x10^3$	9	H
1,500	498	$1.2x10^8$	$9.2x10^3$	13	Days
6,000	1,260	$2.0x10^9$	$2.6x10^4$	20	Months
24,000	3,360	$3.8x10^{10}$	$7.4x10^4$	90	years
96,000	9,218	$7.8x10^{11}$	$2.1x10^5$	5,000	years

Direct Numerical Simulation

- All the effort in a DNS is directed towards the resolution of small scales.
- 99% of the energy is contained outside the dissipation range (the smallest scales).
- Therefore, one thinks of modelling these small scales that have a universal character while fully resolving the larger scales: This would be Large Eddy Simulation.

Large Eddy Simulation

- In LES, the large scales are directly represented while the small scales are modeled using standard modeling techniques (k-e, RSM…)
- We introduce what is called a filter.
- The filter would act as an automation technique that tells the equations what to fully resolve and what to model.

Large Eddy Simulation

- The idea is to decompose the velocity field into a filtered field $\bar{\boldsymbol{U}}(\boldsymbol{x},t)$ and a residual velocity field ***u'(x,t)*** called the residual stress or subgrid scale SGS component.
- Filtering is also characterized by what is called a filter width Δ which defines the smallest size of the eddy to be resolved. All eddies with scales less than Δ are modeled.

Large Eddy Simulation

- Filtering is defined as follows (in one dimension):

$$\bar{\boldsymbol{U}}(\boldsymbol{x},t) = \int_V G(\boldsymbol{r},\boldsymbol{x})\boldsymbol{U}(\boldsymbol{x}-\boldsymbol{r},t)d\boldsymbol{r}$$

Thus, the velocity field has the following decomposition:

$$\boldsymbol{U}(\boldsymbol{x},t) = \bar{\boldsymbol{U}}(\boldsymbol{x},t) + \boldsymbol{u}'(\boldsymbol{x},t)$$

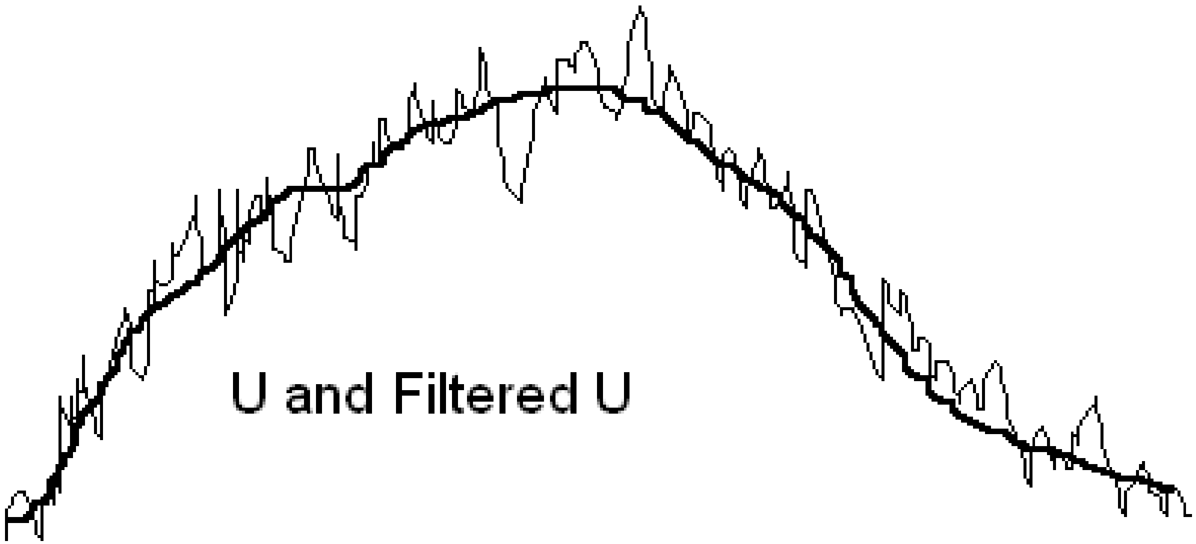
Large Eddy Simulation

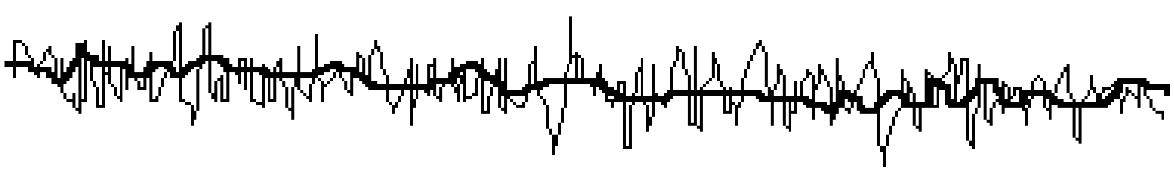
- Previous decomposition is similar to the Reynolds decomposition, however, the terms have totally different meanings.
  - The filtered field is random unlike the average Reynolds field
  - The filtered residual stress is not zero

    $$\overline{\boldsymbol{u}'}(\mathbf{x},t) \neq 0$$

    while the average of the Reynolds stress is zero.

Large Eddy Simulation





u' and Filtered u'

Large Eddy Simulation

- The filtered equations take the following form:
- Continuity Equation:

$$\overline{\left(\frac{\partial U_i}{\partial x_i}\right)} = \frac{\partial \overline{U}_i}{\partial x_i} = 0$$

From which we obtain:

$$\frac{\partial u_i'}{\partial x_i} = \frac{\partial}{\partial x_i}(U_i - \overline{U}_i) = 0$$

Large Eddy Simulation

- The momentum equation is:

$$\frac{\partial \overline{U}_j}{\partial t} + \frac{\partial \overline{U_i U_j}}{\partial x_i} = \nu \frac{\partial^2 \overline{U}_j}{\partial x_i \partial x_i} - \frac{1}{\rho} \frac{\partial \overline{p}}{\partial x_j}$$

We further define the following quantities:

$$\tau_{ij}^R = \overline{U_i U_j} - \overline{U}_i \overline{U}_j \qquad \tau_{ij}^r = \tau_{ij}^R - \tfrac{2}{3} k_r \delta_{ij}$$

$$k_r = \tfrac{1}{2} \tau_{ii}^R \qquad \overline{p} = \overline{p} + \tfrac{2}{3} k_r$$

Large Eddy Simulation

- Finally replacing in the original momentum equation, we get:

$$\frac{\partial \overline{U}_j}{\partial t}+\frac{\partial \overline{U}_i \overline{U}_j}{\partial x_i}=\nu \frac{\partial^2 \overline{U}_j}{\partial x_i \partial x_i}-\frac{\partial \tau_{ij}^r}{\partial x_i}-\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_j}$$

In function of the anisotropic residual stress tensor.

Large Eddy Simulation

- This anisotropic tensor requires modeling in order to close the equations.
- The simplest model is called Smagorinsky model & is an eddy-viscosity model. Modeling takes the following form:

Large Eddy Simulation

$$\tau_{ij}^{r} = -2\nu_r \overline{S}_{ij}$$

Anisotropic Stress Tensor

$$\overline{S}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{U}_i}{\partial x_j} + \frac{\partial \overline{U}_j}{\partial x_i}\right)$$

Filtered Rate of Strain

$$\nu_r = \ell_s^{\,2}\,\overline{\mathsf{S}} = \ell_s^{\,2}\sqrt{2\overline{S}_{ij}\,\overline{S}_{ij}}$$

Eddy viscosity of the residual motions. Modeled using mixing length theory.

$$\ell_s = C_s \Delta$$

Smagorinsky length scale, proportional to the filter width.